

# Dark Matter from the Inflaton Field

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We present a model where inflation and Dark Matter takes place via a single scalar field  $\phi$ . Without introducing any new parameters we are able to unify inflation and Dark Matter using a scalar field  $\phi$  that accounts for inflation at an early epoch while it gives a Dark Matter WIMP particle at low energies. After inflation our universe must be reheated and we must have a long period of radiation dominated before the epoch of Dark Matter. Typically the inflaton decays while it oscillates around the minimum of its potential. If the inflaton decay is not complete or sufficient then the remaining energy density of the inflaton after reheating must be fine tuned to give the correct amount of Dark Matter. An essential feature here, is that Dark Matter-Inflaton particle is produced at low energies without fine tuning or new parameters. This process uses the same coupling  $g$  as for the inflaton decay. Once the field  $\phi$  becomes non-relativistic it will decouple as any WIMP particle, since  $n_\phi$  is exponentially suppressed. The correct amount of Dark Matter determines the cross section and we have a constraint between the coupling  $g$  and the mass  $m_\phi$  of  $\phi$ . The unification scheme we present here has four free parameters, two for the scalar potential  $V(\phi)$  given by the inflation parameter  $\lambda$  of the quartic term and the mass  $m_\phi$ . The other two parameters are the coupling  $g$  between the inflaton  $\phi$  and a scalar field  $\varphi$  and the coupling  $h$  between  $\varphi$  with standard model particles  $\psi$  or  $\chi$ . These four parameters are already present in models of inflation and reheating process, without considering Dark Matter. Therefore, our unification scheme does not increase the number of parameters and it accomplishes the desired unification between the inflaton and Dark Matter for free.

## I. INTRODUCTION

Inflation has now become part of the standard model of cosmology [1]. With new data coming soon, and in particular with the Planck satellite mission, the different inflationary models will need to pass a strong test. Existing observational experiments involve measurement on CMB [4] or large scale structure "LSS" [2] or supernovae SN1a [3], and new proposals are carried out. It has been established that our universe is flat and dominated at present time by Dark Energy "DE" and Dark Matter "DM" with  $\Omega_{DE} \simeq 0.73$ ,  $\Omega_{DM} \simeq 0.27$  and curvature  $\Omega_k \simeq -0.012$  [4]. Inflation sets up the initial perturbations from which gravity forms the large scale structures, including Baryon Acoustic Oscillation, in the universe [2]. Structure formation requires the existence of Dark Matter and therefore the nature and dynamics of inflation and Dark Matter are essential building blocks to understand the current observations.

Inflation is associated with a scalar field, the "inflaton", and the energy scale at which inflation occurs is typically of the order of  $E_I = 10^{16} GeV$  [1] but it is possible to have consistent inflationary models with  $E_I$  as low as  $O(100) MeV$  [5]. On the other hand Dark Matter is described by an energy density which redshifts as  $\rho_{dm} \sim a(t)^{-3}$  and is described by particles where its mass  $m \gg T$ , with  $T$  its temperature. These particles can be either fermions or scalar fields. In the case of scalar fields the scalar potential must be  $V(\phi) = m_\phi^2 \phi^2/2$  and independently on the value of its mass the classical equation of motion ensures that  $\rho_\phi \sim V \sim a(t)^{-3}$ .

Here, since we want to unify inflation with Dark Matter we will assume that DM is made out of the same

field which we denote by  $\phi$ . A scalar field can easily give inflation and DM if the potential is flat at high energies and at low energies the potential approaches the limit  $V(\phi) \rightarrow m_\phi^2 \phi^2/2$  [13]. However, most of the time our universe was dominated by radiation. Therefore, any realistic model must not only explain the two stages of inflation and Dark Matter but must reheat the universe and allow for a long period of radiation domination. Typically the inflaton decays while it oscillates around the minimum of its potential [1]. If the inflaton decay is not complete or efficient then the remaining energy density of the inflaton after reheating must be fine tuned to give the correct amount of Dark Matter [6].

To reheat the universe we couple  $\phi$  to a relativistic field  $\varphi$  via an interaction term  $L_{int}$ . This field  $\varphi$  may be a standard model "SM" particle, as for example neutrinos, but it could also be an extra relativistic particle not contained in the SM. Here we will assume for simplicity that  $\varphi$  is an auxiliary scalar field and extra relativistic degrees of freedom are fine with the cosmological data [7]. After inflation the interaction term between  $\phi$  and  $\varphi$  that we use is the standard  $V_{int} \sim g^2 \phi^2 \varphi^2/2$ . This process has been widely studied [8–11] and we know that after a period of preheating and parametric resonance the term  $V_{int}$  dominates over the inflationary potential  $V(\phi)$  and one produces  $\varphi$  particles with  $n_\varphi \approx n_\phi$ , and there is then not a complete decay of  $\phi$  [8] and has also been confirmed numerically calculated in lattice [11]. To achieve an efficient decay one needs to couple  $\phi$  or  $\varphi$  to other fields using a trilinear term [9, 10]. We introduce a trilinear term  $h \varphi \bar{\psi} \psi$  between  $\varphi$  and fermion fields  $\psi$  and we obtain an efficient decay for  $\varphi$  and  $\phi$  which stops at  $\Omega_\phi \approx 10^{-23}$ . At a much later stage we show that the fields  $\varphi$  and  $\phi$  are produced using the same interaction

terms as for the reheating process. This is a main difference between other unification schemes where one needs to fine tune the amount of Dark Matter at the end of reheating and initial radiation domination epoch. However in our scheme the regeneration of  $\phi$  takes place naturally without fine tuning and, once the field  $\phi$  becomes non-relativistic, when its mass  $m_o > T$ ,  $\phi$  will decouple since  $n_\phi$  is exponentially suppressed. The correct amount of Dark Matter determines the cross section of Dark Matter at decoupling (as any WIMP particle) giving a constraint between the coupling  $g$  and the mass  $m_o$ .

We will work out the inflation-Dark Matter unification through a simple example and we show in Fig.1 the evolution of the energy densities  $\rho_\phi$  and radiation. Of course, the whole scheme is much more general and other inflaton-Dark Matter potentials  $V(\phi)$  or interaction terms may be used, however in all cases  $V(\phi) \rightarrow m_o^2 \phi^2/2$  at late times to have Dark Matter. The unification scheme we present here has four free parameters, two for the scalar potential  $V(\phi)$  given by the inflation parameter  $\lambda$  of the quartic term and the mass  $m_o$  given by the quadratic term in  $V(\phi)$ . The other two parameters are the coupling  $g$  between the inflaton  $\phi$  and a scalar field  $\varphi$  and the coupling  $h$  between  $\varphi$  with standard model particles  $\psi$  or  $\chi$ . Density perturbations normalized to COBE fixes the value for  $\lambda$  and the correct amount of Dark Matter determines the cross section of Dark Matter at decoupling (WIMP particle) gives a constraint between  $g$  and  $m_o$ , leaving only the mass of the Dark Matter particle  $m_o$  as a free parameter. However there is a lower limit for  $m_o$  from Hot and Warm Dark Matter with  $m_o \geq O(10 \text{ KeV})$  [14] and an upper limit  $m_o \ll E_h \simeq 10^6 \text{ GeV}$ , so that  $\phi$  may be produced. We will show that the same couplings that give the inflaton decay gives the Dark Matter regeneration at low scales and sets in combination with  $m_o$  the WIMP decoupling cross section. These four parameters are already present in models of inflation and reheating process [8]. This implies that our unification scheme does not increase the number of parameters and it accomplishes the desired unification between inflaton and Dark Matter for free.

We organize the present work as follows: In Sec.(II) we present the general framework, including the scalar potential  $V$  and the interaction terms. In Sec.(III) we discuss the different interaction terms for the inflaton decay, the regeneration of the fields  $\psi$  and  $\phi$  at a much later time and the decoupling of  $\phi$  as a WIMP particle. Finally we give the summary and conclusion in Sec.(IV).

## II. GENERAL FRAMEWORK

Our starting point is a flat FRW universe with the inflaton-dark matter field  $\phi$  coupled to a scalar  $\varphi$ . We take the lagrangian  $L = L_{SM} + \tilde{L}$ , where  $L_{SM}$  is the

standard model SM lagrangian,

$$\tilde{L} = \frac{1}{2} \partial_\mu \phi \partial^\mu \phi + \frac{1}{2} \partial_\mu \varphi \partial^\mu \varphi - V(\phi) - B(\varphi) + L_{int}(\phi, \varphi, SM), \quad (1)$$

$V(\phi), B(\varphi)$  are the scalar potentials for  $\phi, \varphi$  and  $V_{int} = -L_{int}$  is the interaction potential. The classical evolution of  $\phi$  and  $\varphi$  are given by the equations of motion,

$$\ddot{\phi} + 3H\dot{\phi} + V' + V'_{int} = 0 \quad (2)$$

$$\ddot{\varphi} + 3H\dot{\varphi} + B_\varphi + V_{int,\varphi} = 0 \quad (3)$$

with  $H^2 \equiv (\dot{a}/a)^2 = \rho/3$ , a prime denotes derivative w.r.t.  $\phi$ ,  $B_\varphi \equiv \partial B/\partial \varphi$  and we take natural units  $m_{pl}^2 = 1/8\pi G \equiv 1$ .

There are many potentials  $V(\phi)$  that lead to an inflation epoch at early times and as Dark Matter at late times. Inflation with a single scalar field can be classified in small or large field models [? ]. Small fields are potentials that inflate for values of the inflaton  $\phi \ll m_{pl}$  as in new inflation models, e.g.  $V = V_o(\phi^2 - \mu^2)^2$ , while large field models are when inflation occurs for  $\phi > m_{pl}$  as in chaotic models, e.g.  $V = V_i \phi^n$ . Here we will assume the simplest chaotic potential because we are interested in showing how the inflaton-Dark Matter unification scheme takes place in the context of  $\phi$  regeneration process. However, more general potentials and interaction terms may be used in the context of Dark Matter regeneration scheme presented here.

We take as the inflaton-Dark Matter potential

$$V(\phi) = \frac{1}{2} m_o^2 \phi^2 + \frac{1}{4} \lambda \phi^4 \quad (4)$$

and we will assume a massless  $\varphi$ , with  $B(\varphi) = 0$ , so that  $\varphi$  may have only an effective mass due to its interactions with other fields. The interaction term that we choose is as standard four leg between  $\phi$  and  $\varphi$ , which has been widely studied in the literature in the context of reheating and preheating [8] and we also couple  $\varphi$  to standard model "SM" particles  $\chi, \psi$ , since at high energies the universe must be dominated by SM particles. The interaction we use is

$$V_{int} = \frac{1}{2} g^2 \phi^2 \varphi^2 + \frac{1}{2} h^2 \varphi^2 \chi^2 + h \varphi \bar{\psi} \psi \quad (5)$$

where  $g, h$  are constant couplings. The mass of the different fields are

$$m_\phi^2 = V''(\phi) + V''_{int} = m_o^2 + 3\lambda \phi^2 + g^2 \varphi^2, \quad (6)$$

$$m_\varphi^2 = \frac{d^2 B}{d\varphi^2} + \frac{d^2 V_{int}}{d\varphi^2} = g^2 \phi^2 \quad (7)$$

$$m_\chi^2 = h^2 \varphi^2 \quad (8)$$

$$m_\psi = h \varphi \quad (9)$$

and are in general field and time dependent quantities and we took  $B(\varphi) \equiv 0$ . We will discuss the phenomenology of eqs.(4) and (5) in Sec.III.

### 1. Inflaton Potential $V(\phi)$

The potential  $V(\phi)$  must satisfy at the inflation scale the slow roll conditions  $|V'/V| < 1$ ,  $|V''/V| < 1$  and the constraint on the energy density perturbation normalized to COBE [1]

$$\frac{\delta\rho}{\rho} = \frac{1}{\sqrt{75}\pi^2} \frac{V^{3/2}}{V'} = 1.9 \times 10^{-5}. \quad (10)$$

For example for the chaotic potential  $V_4 \equiv \lambda\phi^4$  inflation occurs for  $\phi > \phi_e = \sqrt{8}m_{pl}$  and the dimensionless parameter  $\lambda$  and the scale of inflation  $E_I \equiv V(\phi_e)^{1/4}$  are

$$V_4 \equiv \frac{1}{4}\lambda\phi^4, \quad (11)$$

$$\lambda = 2.5 \times 10^{-13}, \quad E_I = 3 \times 10^{15} GeV. \quad (12)$$

If we take  $V_2 \equiv m_o^2\phi^2/2$  as the inflationary potential then one has  $E_I = 6 \times 10^{15} GeV$  with a mass  $m_o \simeq 10^{13} GeV$ . However, since we are interested that  $\phi$  accounts for Dark Matter at low energies we need a much smaller mass  $m_o \ll 10^{13} GeV$  (c.f. eq.(41)), since we have to produce them for  $T < E_h \simeq 10^6 GeV$ , and with  $m_o = O(GeV)$  the quadratic term is subdominant compared to the quartic term in the potential in eq.(4) at the inflationary epoch.

### 2. Dark Matter Potential $V(\phi)$

The field  $\phi$  should give us Dark Matter at low energies, and its energy density  $\rho_\phi$  must then redshift as  $a^{-3}$ , with an equation of state "EOS"  $w_\phi = 0$ . A scalar field with only gravitational interactions with other fields and with a potential  $V(\phi) = V_i\phi^n$  redshifts as  $a^{-3(1+w)}$  with  $w = (n-2)/(n+2)$  (for  $n$ -even) and only for  $n = 2$  do we have a energy density redshifting as matter,  $V_2 \propto \phi^2 \propto a^{-3}$  while for  $V_4 \propto \phi^4 \propto a^{-4}$  and  $w = 1/3$  [13]. Therefore, the scalar potential  $V(\phi)$  must have the limit at low energies

$$V(\phi) \rightarrow V_2 \equiv \frac{1}{2}m_{\phi o}^2\phi^2 \quad (13)$$

with  $m_o$  a constant mass term as in eq.(4). The constraint on  $V_2$  is that at present time (from here on the subscript  $o$  gives present time quantities)

$$\rho_\phi(t_o) = \frac{1}{2}\dot{\phi}_o^2 + V_2(t_o) = 2V_2(t_o) = m_o^2\phi_o^2 = \rho_{dmo} \quad (14)$$

where we used that the pressure vanishes  $p = \dot{\phi}^2/2 - V_2 = 0$  and  $\rho_{dmo}$  is the present time Dark Matter density. At low energies the mass of the Dark Matter particle is given by  $m_o$  and CDM requires a mass of  $\phi$  to be  $m_o \geq O(GeV)$  while warm DM has a smaller mass with  $m_o > O(10 - 100) keV$  [14].

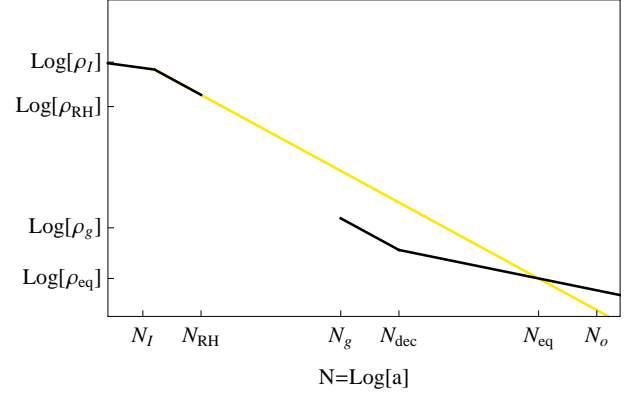


FIG. 1: We show the evolution of the inflaton-Dark Matter field  $\phi$  (black) and of the relativistic (yellow) energy densities. Notice that between  $N_{RH}$  and  $N_{gen}$  there is no  $\rho_\phi$  (no  $\phi$  particles) and that the evolution of  $\rho_\phi$  goes from relativistic to Dark Matter type at  $N_{dec}$ .

### 3. Potentials $V_2 = V_4$

The potential  $V(\phi) = V_2 + V_4 = \frac{1}{2}m_o^2\phi^2 + \frac{1}{4}\lambda\phi^4$  is dominated by the quartic term if the oscillations of  $\phi$  are larger than

$$\phi \geq \phi_{24} \equiv \sqrt{\frac{2}{\lambda}} m_o = 2.8 \times 10^6 m_o \quad (15)$$

and we define the epoch where  $V_2(\phi) = V_4(\phi)$  at  $\phi = \phi_{24}$  and  $V_{24} = V_2 = V_4$  with

$$E_{24} \equiv V_2(\phi_{24})^{1/4} = \left( \frac{m_o^2\phi_{24}^2}{2} \right)^{1/4} = \frac{m_o}{\lambda^{1/4}} = 1.4 \times 10^3 m_o \quad (16)$$

where we have taken  $\lambda = 2.5 \times 10^{-13}$  from eq.(12) in the r.h.s. of eqs.(15) and (16). The potential  $V_4$  dominates at high energies, during inflation, while  $V_2$  at low energies, when Dark Matter prevails.

### III. INTERACTION TERM $V_{int}(\phi, \varphi, SM)$

The aim of this section is to show how a single scalar field  $\phi$  can account for inflation at early times and as Dark Matter at a late time, without any fine tuning of the parameters. In order to achieve this we must have a sufficient decay of the  $\phi$  field to reheat the universe with SM particles and at a much later stage to regenerate  $\rho_\phi$  such that it gives the Dark Matter today. We will show that the interaction term in eq.(5) does indeed allow for a sufficient decay of  $\phi$  at high energies, but at the same time it will allow for generating  $\rho_\phi$  at much lower energies without the need to introduce any extra parameters. The generation of  $\rho_\phi$  then allows to have  $\phi$  as Dark Matter. By sufficient decay we mean that the universe at the end of reheating and the beginning of radiation dominated

universe has at most a remanent of  $\rho_\phi$  which is subdominant compared to the amount of Dark Matter given by  $\rho_{dm} = \rho_{dmo}(a_o/a)^3$  at that time.

We will now discuss the different interactions and decays but first we present our definition of sufficient decay in more detail.

### A. Sufficient Decay

The inflaton decay can have a complete decay into other particles giving a  $\rho_\phi \simeq 0$  after reheating. However, we may have a remanent of  $\rho_\phi$  which may or may not be cosmologically relevant, depending on the value of  $\Omega_\phi$ .

Since the reheating process gives a radiation dominated universe the dominate energy density is  $\rho \sim a^{-4}$  while a remanent of  $\phi$  has  $\rho_\phi \sim a^{-3}$ , taking that  $V_2$  dominates over  $V_4$  and that the interaction is now subdominant, i.e. for  $\phi \ll \phi_{24}$  and the evolution of a non-interacting fluid with  $V_2$  is  $\rho_\phi \sim a^{-3}$ .

The amount of Dark Matter at a scale  $E_i = E_o(a_o/a_i)$ , with  $E \equiv \rho^{1/4}$ , is given by

$$\begin{aligned} \frac{\Omega_{dmi}}{\Omega_{ri}} &= \frac{\rho_\phi}{\rho_r} = \frac{\rho_{\phi o}}{\rho_{ro}} \left( \frac{a_d}{a_o} \right) = \frac{\Omega_{\phi o}}{\Omega_{ro}} \left( \frac{E_o}{E_d} \right) \\ &= 6.3 \times 10^{-23} \left( \frac{10^{13} \text{GeV}}{E_i} \right) \end{aligned} \quad (17)$$

with  $\Omega_{\phi o} = \Omega_{dmo} = 0.22$ ,  $\Omega_{ro} h^2 = 4.15 \times 10^{-5}$  the present time relativistic energy density and  $E_o = 2.4 \times 10^{-13} \text{GeV}$ . If after reheating we have a reheating energy  $E_i$  with  $\Omega_\phi(E_i) \ll \Omega_{dmi}$  then we say the decay was sufficient. A sufficient decay would clearly not allow  $\rho_\phi$  to account for Dark Matter since it would not give the correct amount of DM. Of course having  $\Omega_\phi(E_i) = \Omega_{dmi}$  giving the exact amount of Dark Matter at a high energy  $E_i$  would require a fine tuning in the parameters of the model and we want to avoid this scenario.

We can estimate the amount of Dark Matter required at  $E_i = E_{24} = m_o/\lambda^{1/4}$  and using eq.(17) we find

$$\Omega_{dm}(E_{24}) = 4.5 \times 10^{-13} \left( \frac{\text{GeV}}{m_o} \right) \quad (18)$$

where we have taken eq.(12) and  $\Omega_{ri} \simeq 1$ . If the  $\phi$  field is present at energies  $E_{24}$  its classical evolution requires that  $\Omega_\phi$  is fine tuned to given the value in eq.(18) to account for Dark Matter, unless it is produced again at a later stage.

### B. Inflaton Decay and Universe Reheating

Let us now discuss the interaction term  $g^2 \phi^2 \varphi^2/2$  between  $\phi$  and  $\varphi$  with a scalar potential  $V(\phi)$  given in eq.(4). This interaction has been widely studied in the context of preheating and reheating [8]. We will present

here the main results and we refer to the original work for details [8]. Soon after inflation, the inflaton field  $\phi$  oscillates with large amplitude  $\phi \gg \phi_{24}$  and the  $\lambda \phi^4$  term in the potential  $V$  dominates. The first stage of reheating is a period of preheating and parametric resonance with  $\phi$  producing  $\varphi$  particles in a very efficient process and in a short amount of time [8]. However, the back reaction of  $\varphi$  is important and one ends up with an incomplete decay of  $\phi$  [8]. In fact it is the term  $g^2 \phi^2 \varphi^2/2$  that dominates over  $V_4 = \lambda \phi^4/4$ , since the coupling  $\lambda \simeq 10^{-13} \ll g^2$ , and one ends with  $n_\phi \approx n_\varphi$  and  $\phi^2 \approx \varphi^2$ . So, taking only the interaction term  $g^2 \phi^2 \varphi^2/2$  we cannot achieve a complete decay of  $\phi$  [8].

In order to have a complete decay one can introduce trilinear terms such as  $h^2 \sigma \phi \chi^2$  or  $h \phi \bar{\psi} \psi$ , with  $\sigma$  an energy scale which can be related to the v.e.v. of the field  $\phi$  [9, 10]. These trilinear terms do produce a complete decay of  $\phi$  and we will end up with a universe dominated by SM particles (if  $\chi, \psi$  are SM particles or coupled to the SM). The dominant process at late time is the perturbative decay of  $\phi \rightarrow \psi \psi$  into fermions which dominates over the scattering  $\phi \phi \rightarrow \varphi \varphi$  [9, 10]. This decay takes place as long as the decay rate  $\Gamma_{\phi \psi \psi} = h^2 m_\phi / 8\pi$  is larger than  $H$  and the mass of  $\phi$  satisfies  $m_\phi > \sqrt{2} m_\psi = \sqrt{2} h |\phi|$  where  $m_\phi$  is the mass of the scalar field. Even though, we can reheat the universe with the trilinear term between  $\phi$  and the fermions  $\psi$ , this same term will not allow  $\phi$  to play the roll of Dark Matter at late times. This is because the mass of  $\phi$  at low energies is given by the constant  $m_o$  and we would then have a constant decay rate  $\Gamma_{\phi \psi \psi} = h^2 m_o / 8\pi$ , but since  $H$  decreases, we will have  $\Gamma_{\phi \psi \psi} / H \gg 1$  and the decay will not stop and  $\phi$  will decay completely.

In order to avoid a complete decay of  $\phi$  we choose to couple  $\varphi$  to fermions  $\psi$  with a trilinear term  $h \varphi \bar{\psi} \psi$  as in eq.(5). In this case it will be  $\varphi$  decaying into  $\bar{\psi} \psi$  with a decay rate

$$\Gamma_{\varphi \psi \psi} = \frac{h^2 m_\varphi}{8\pi} \quad (19)$$

and masses  $m_\varphi = g|\phi|$  and  $m_\psi = h|\varphi|$ . The trilinear term  $h \varphi \bar{\psi} \psi$  allows  $\varphi$  to decay into fermions  $\psi$  in a perturbative process if the mass condition  $m_\varphi^2 = g^2 \phi^2 > 2m_\psi^2 = 2h^2 \varphi^2$  and  $\Gamma_{\phi \psi \psi} / H > 1$  are satisfied. The mass condition gives the constraint  $2h^2 \leq g^2$  if we take  $\phi^2 \approx \varphi^2$ . The decay of  $\varphi$  will then drag the inflaton field  $\phi$  in the chain reaction  $\phi \leftrightarrow \varphi \rightarrow \bar{\psi} + \psi$ . The decay rate is

$$\left( \frac{\Gamma_{\varphi \psi \psi}}{H} \right)^2 = \frac{3h^4 m_\varphi^2 m_{pl}^2}{64\pi^2 \rho} \equiv \frac{\rho_R}{\rho} \quad (20)$$

where we have set  $3H^2 m_{pl}^2 = \rho$  and we define

$$\rho_R \equiv \frac{3h^4 m_\varphi^2 m_{pl}^2}{64\pi^2} \quad (21)$$

and  $\Gamma_{\varphi \psi \psi} / H \geq$  requires  $\rho < \rho_R$  and therefore  $\rho_R$  sets the initial stage of the decay. However,  $\rho_R$  is a function of



$m_\phi^2(\phi)$ . Let us first consider the case when  $V_4$  dominates the potential  $V$ , i.e. for  $\phi \gg \phi_{24}$ . In this case we have  $m_\phi^4 = g^4 \phi^4 = 4g^4 V_4/\lambda$  and setting  $V_4 = \rho_\phi(1 - w_\phi)/2 \sim \rho_\phi/3$ ,  $w_\phi \sim 1/3$  we have

$$\left(\frac{\Gamma_{\varphi\psi\psi}}{H}\right)^4 = \frac{3h^8 g^4 m_{pl}^4 \rho_\phi}{1024\pi^4 \lambda \rho^2} = \Omega_\phi \frac{\rho_{Ri}}{\rho} \quad (22)$$

with  $\Omega_\phi = \rho_\phi/\rho$  and

$$\rho_{Ri} \equiv \frac{3h^8 g^4 m_{pl}^4}{1024\pi^4 \lambda} \simeq \left(\frac{2h^2}{g^2}\right)^4 \left(\frac{m_o}{GeV}\right)^6 (2.5 \times 10^{14} GeV)^4 \quad (23)$$

and we have used  $2h^2 = g^2$  and eq.(41) in last expression of eq.(23). We have a decay if  $\Gamma_{\varphi\psi\psi}/H > 1$  which implies a lower limit for  $\Omega_\phi$  given by  $\Omega_\phi > \rho/\rho_{Ri}$ . Clearly since  $\Omega_\phi \leq 1$  the upper limit to the initial decay rate is given by  $\rho_{Ri}$  with a decay energy  $E_{Ri} \equiv \rho_{Ri}^{1/4} \simeq (2h^2/g^2)(m_o/GeV)^{1/6} 10^{14} GeV$ . The eq.(22) gives a balance between  $\rho_\phi$  and  $\rho$  with  $\rho_\phi \simeq \rho^2/\rho_{Ri}$  and  $\Gamma_{\varphi\psi\psi}/H \simeq 1$ , which implies that  $\Omega_\phi$  dilutes as  $\rho$  and the decay does not stop. If  $\rho_\phi$  dilutes slower than  $\rho^2$  than  $\Gamma_{\varphi\psi\psi}/H > 1$  giving a decay but if  $\rho_\phi$  dilutes faster than  $\rho^2$  than  $\Gamma_{\varphi\psi\psi}/H < 1$  stopping the decay.

So, let us consider now the case when  $V_2$  dominates  $V$ . In this case we can use  $m_\varphi^2 = g^2 \phi^2 = 2g^2 V_2/m_o^2$  with  $\rho_\phi = 2V_2$ , and eq.(20) becomes

$$\left(\frac{\Gamma_{\varphi\psi\psi}}{H}\right)^2 = \frac{3h^4 g^2 m_{pl}^2 \rho_\phi}{64\pi^2 m_o^2 \rho} = \frac{\Omega_\phi}{\Omega_{\phi Rf}} \quad (24)$$

where we defined

$$\Omega_{\phi Rf} \equiv \frac{64\pi^2 m_o^2}{3g^2 h^4 m_{pl}^2} = \left(\frac{g^2}{2h^2}\right)^2 \left(\frac{GeV}{m_o}\right) 3 \times 10^{-23} \quad (25)$$

and we have take a  $2h^2 = g^2$  and eq.(41) to evaluate the last expression in eq.(25). Notice that  $\Omega_{\phi Rf}$  is independent of  $\phi$  and  $\rho$ , so once the ratio  $\Omega_\phi$  becomes smaller than  $\Omega_{\phi Rf} \approx 10^{-23}$  the decay stops. We can estimate the energy at the end of the decay given by  $\rho_{Rf} = \rho_\phi/\Omega_{\phi Rf}$  and if we take the upper limit for  $\rho_\phi = 2V_2 = 2V_{24}$  we have

$$\rho_{Rf} = \frac{3g^2 h^4 m_o^2 m_{pl}^2}{32\lambda\pi^2} \simeq \left(\frac{g^2}{2h^2}\right)^2 \left(\frac{m_o}{GeV}\right)^5 (6.8 \times 10^8 GeV)^4 \quad (26)$$

We conclude that range of energies where the  $\varphi$  decay takes place is given by the upper limit  $E_{Ri} = \rho_{Ri}^{1/4}$  and the lower limit  $E_{Rf} = \rho_{Rf}^{1/4}$  given by eqs.(23) and (26), i.e.

$$E_{Ri} \equiv \rho_{Ri}^{1/4} = \left(\frac{2h^2}{g^2}\right) \left(\frac{m_o}{GeV}\right)^{1/6} 2.5 \times 10^{14} GeV \quad (27)$$

$$E_{Rf} \equiv \rho_{Rf}^{1/4} = \sqrt{\frac{2h^2}{g^2}} \left(\frac{m_o}{GeV}\right)^{5/4} 6.8 \times 10^8 GeV. \quad (28)$$

We can compared eq.(25) with the amount of Dark Matter in eq.(17) for different energies. For example for

$E_i = E_{Ri} \simeq 10^{14} GeV$  we have  $\Omega_{dm}(E_{Ri}) = 6 \times 10^{-24} < \Omega_{\phi Rf} \approx 10^{-23}$  and following the discussion below eq.(23), as long as  $V_4$  dominates there is decay, but as soon as  $V$  is dominated by  $V_2$  the decay will stop since  $\Omega_\phi \leq \Omega_{\phi Rf}$ . From eq.(25) for  $E_i = E_{Rf} \simeq 10^9 GeV$  we have  $\Omega_{dm}(E_{Rf}) = 6 \times 10^{-19}$  which is much larger than  $\Omega_{\phi Rf} \approx 10^{-23}$ . We see that the decay rate  $\Gamma_{\varphi\psi\psi}$  gives a sufficient decay of  $\phi$  and if we want to have  $\rho_\phi$  as Dark Matter the field  $\phi$  must be produced at a later stage.

### C. Production of $\phi$ and $\varphi$ at low scales

If the field  $\phi$  decays completely (or sufficiently) after inflation then there will no residual  $\rho_\phi$  left to account for Dark Matter, as seen in then last section in eqs.(18) and (25). In order to produce the field  $\phi$  at lower energies we use the same interaction terms given in eq.(5), i.e.  $g^2 \phi^2 \varphi^2/2$  and  $h\varphi\bar{\psi}\psi$ , as for the inflaton decay but at a very different energy scale and where the properties of the fields are different.

We have seen in Sec.III B that the reheating scale is of the order of  $T_R \simeq 10^9 - 10^{12} GeV$ , the fields  $\varphi$  does no longer decay into  $\psi$  and that the fields  $\varphi$  and  $\phi$  are subdominant, at best. So we expect that at low scales  $T \ll T_R$ , but still above  $T \gg TeV$  and  $m_o$ , that the particles  $\varphi, \phi, \psi$  are all relativistic. This is certainly true for all standard model particles and since the mass of  $\varphi$  depends on the value of  $\phi$ ,  $m_\varphi^2 = h^2 \phi^2 \simeq 0$  for  $\rho_\phi \sim \phi^2 \simeq 0$ . When the particles  $\psi, \varphi$  and  $\phi$ , are relativistic the interaction terms in eq.(5) give  $2 \leftrightarrow 2$  scattering processes. The interaction rate for the  $2 \leftrightarrow 2$  perturbative scattering process for relativistic particles is

$$\Gamma_{22} = \frac{|M_{ab}|^2 n_a}{32\pi^2 E_a^2} = \langle \sigma v \rangle n_a. \quad (29)$$

where  $|M_{ab}|^2$  is the transition amplitude of the process,  $E_a$  the energy of the incident particle and  $\langle \sigma v \rangle$  is the cross section times the relative velocity  $v$  of the initial particles with

$$\langle \sigma v \rangle = \frac{|M_{ab}|^2}{32\pi^2 E_a^2}. \quad (30)$$

Since the particles are relativistic the number density is given by  $n_a = g_a \zeta[3] \pi^2 T^3/30 = c_n E^3$  with  $c_n = g_a \zeta(3)/\pi^2 \bar{r}^3$ ,  $E = \bar{r}T$  and  $\bar{r} \equiv (\rho/nT) = \pi^4/30\zeta(3) \simeq 2.7$  and eq.(29) becomes

$$\Gamma_{22} = c_{22} |M_{ab}|^2 E_a \quad (31)$$

with  $c_{22} = \zeta(3)/(32\pi^3 \bar{r}^3)$ . In the case of the interaction term  $h\varphi\bar{\psi}\psi$  the  $2 \leftrightarrow 2$  process given by  $\psi + \psi \leftrightarrow \varphi + \varphi$  has  $|M_{ab}|^2 = h^4$  and

$$\Gamma_{\psi\psi\varphi\varphi} = c_{22} h^4 E. \quad (32)$$

This interaction takes place if  $\Gamma_{\psi\psi\varphi\varphi} > H$  and taking  $H = \sqrt{\rho_r/(3m_{pl}^2\Omega_r)} \equiv c_H E^2/m_{pl}$  we have

$$\frac{\Gamma_{\psi\psi\varphi\varphi}}{H} = \frac{c_d h^4 m_{pl}}{E} \equiv \frac{E_h}{E}, \quad (33)$$

$$E_h \equiv c_d h^4 m_{pl} = \left(\frac{2h^2}{g^2}\right)^2 \left(\frac{m_o}{GeV}\right)^2 2 \times 10^6 GeV \quad (34)$$

where  $c_d \equiv c_{22}/c_H$ ,  $c_H^2 \equiv g_r \pi^2/(90\Omega_r \bar{r}^4)$  and we have take a  $2h^2 = g^2$  and eq.(41) to evaluate the last expression in eq.(34). We see from eq.(33) that as long as all particles involved, (i.e.  $\psi, \varphi$ ), are relativistic and  $E < E_h$  we will produce relativistic  $\varphi$  particles and  $\psi$  and  $\varphi$  will be in thermal equilibrium for  $E \leq E_R$ . For  $E > 10^2 GeV$  we have  $g_r \simeq 106$ ,  $\Omega_r \simeq 1$  and  $c_d \simeq 10^{-4}$ .

At the same time, once the relativistic field  $\varphi$  has been produced the interaction term  $g^2 \phi^2 \varphi^2/2$  that couples  $\phi$  to  $\varphi$  will produce relativistic  $\phi$  particles as long as  $T \gg m_o$ . This interaction term also gives a  $2 \leftrightarrow 2$  perturbative process and from eq.(31) with  $|M_{ab}|^2 = g^4$  we have

$$\Gamma_{\varphi\varphi\phi\phi} = c_{22} g^4 E, \quad H = \sqrt{\frac{\rho_r}{3m_{pl}^2\Omega_r}} = c_H E^2, \quad (35)$$

$$\frac{\Gamma_{\varphi\varphi\phi\phi}}{H} = \frac{c_d g^4 m_{pl}}{E} \equiv \frac{E_g}{E} \quad (36)$$

with  $c_d \equiv c_{22}/c_H$  as in eq.(33). The process takes place when  $\Gamma_{\varphi\varphi\phi\phi}/H > 1$  or

$$E \leq E_g \equiv c_d g^4 m_{pl} = \left(\frac{m_o}{GeV}\right)^2 8 \times 10^6 GeV \quad (37)$$

and we have take eq.(41) to evaluate the last expression in eq.(37). For  $E < E_g, E_h$  as long as all the particles are relativistic they are in thermal equilibrium with and  $T_\phi = T_\varphi \simeq T_\psi$ . From eqs.(33) and (37) we have  $E_h = (h^4/g^4)E_g \leq E_g/4$  since  $2h^2 \leq g^2$ . Therefore, the field  $\phi$  is produced as soon as  $\varphi$  are produced. As long as  $\phi$  is relativistic, i.e.  $T \gg m_o$ , we will  $\Omega_\varphi = \Omega_\phi$ , but once we reach the region with  $T \approx m_o$  the two fields will decouple since  $n_\phi$  will be exponentially suppressed.

#### D. Non-relativistic $\phi$ Decoupling: $E_{dec}$

If two relativistic particles ( $\varphi, \phi$ ) are in thermal equilibrium and one (in our case  $\phi$ ) becomes non-relativistic then the density number  $n_\phi$  is exponentially suppressed and  $\varphi$  and  $\phi$  decouple. This is just the standard WIMP particle decoupling. The transition rate for a  $2 \leftrightarrow 2$  process is given by eq.(29)

$$\Gamma_{dec} = \langle \sigma v \rangle n_\phi. \quad (38)$$

In order to have the correct amount of Dark Matter a WIMP must decouple with a  $\langle \sigma \rangle$  such that [12]

$$\Omega_{\phi o} h_o^2 = \frac{3 \times 10^{-27} cm^3 s^{-1}}{\langle \sigma v \rangle}. \quad (39)$$

For the  $2 \leftrightarrow 2$  transition with  $\Omega_{\phi o} \simeq 0.22, h_o = 0.7$  eq.(39) implies a cross section

$$\langle \sigma \rangle = \frac{g^4}{32\pi m_o^2} = 0.1 pb \quad (40)$$

with  $pb = 10^{-36} cm^2 = (5 \times 10^{-5}/GeV)^2$  and  $v \simeq c$ , giving a value for  $g^2$

$$g^2 = 1.6 \times 10^{-4} \left(\frac{m_o}{GeV}\right). \quad (41)$$

Eq.(40) gives a constraint for  $g$  in terms of the mass  $m_o$ . The freeze out takes place at  $x_F = m_o/T_F \simeq 10$  [12] giving a decoupling constant energy  $E_{dec}$  with

$$E_{dec} = c_{dec} T_F = c_{dec} \frac{m_o}{x_F} \simeq 0.12 m_o, \quad (42)$$

and  $c_{dec} = (\pi^2 g_{rel}/30)^{1/4} \simeq 1$  taking  $g_{rel} = g_{sm} + g_\varphi = 5.5 + 1 = 6.5$  at  $E < O(MeV)$ . For energies  $E < E_{dec}$  the fields  $\phi$  and  $\varphi$  are no longer coupled and  $\phi$  evolves classically as matter with  $\rho_\phi = \rho_{\phi o}(a_o/a)^3$ . The constraint on warm Dark Matter sets a lower scale  $m_o > O(10 - 100 keV)$  [14].

#### 1. Dark Matter Decay?: $E_{Ddm}$

We have seen that at  $E_{dec}$  the field  $\phi$  ceases to maintain thermal equilibrium with  $\varphi$  through the  $2 \leftrightarrow 2$  process. However, the field  $\phi$  may decay into  $\varphi$  since  $m_\phi \gg m_\varphi$ . Of course, we do not want  $\phi$  to decay since it must account for Dark Matter. In this case the decay  $\phi \rightarrow \varphi\varphi$  has a decay rate  $\Gamma_{\phi\varphi\varphi}$  for the interaction term  $g^2 \phi^2 \varphi^2/2$  is

$$\Gamma_{\phi\varphi\varphi} = \frac{g^4 \phi^2}{8\pi m_o}. \quad (43)$$

Using that  $\phi^2 = 2V_2/m_o^2 = \rho_\phi/m_o^2$  and  $H^2 = \rho_\phi/3m_{pl}^2$ , in matter dominated region, we have

$$\left(\frac{\Gamma_{\phi\varphi\varphi}}{H}\right)^2 = \frac{g^8 \rho_\phi^2}{64\pi^2 m_o^6 H^2} = \frac{3g^8 \rho_\phi}{64\pi^2 m_o^6} \quad (44)$$

and  $\Gamma_{\phi\varphi\varphi}/H < 1$  if

$$\rho_\phi < \frac{64\pi^2 m_o^6}{3g^4} = \left(\frac{m_o}{GeV}\right)^{2/3} (3.6 \times 10^{13} GeV)^4 \quad (45)$$

where we have used eq.(41) on the r.h.s. of eq.(45). Eq.(45) clearly shows that  $\phi$  will not decay into  $\varphi$  at energies  $E \ll 10^{13} GeV$  where Dark Matter is relevant.

## IV. SUMMARY AND CONCLUSIONS

We have presented a model where inflation and Dark Matter takes place via a single scalar field  $\phi$ . The unification scheme presented here has four free parameters,

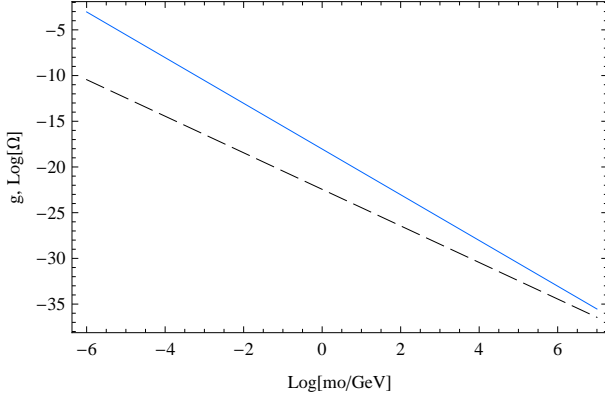


FIG. 2: We show the dependence of  $\Omega_{dm}(E_{Rf})$  and  $\Omega_{\phi}(E_{Rf})$  (blue, dashed-black, respectively) at  $E_{24}$  as a function of  $m_o$ . We notice that  $\Omega_{dm}(E_{Rf}) \gg \Omega_{\phi}(E_{Rf})$  giving a sufficient inflaton decay.

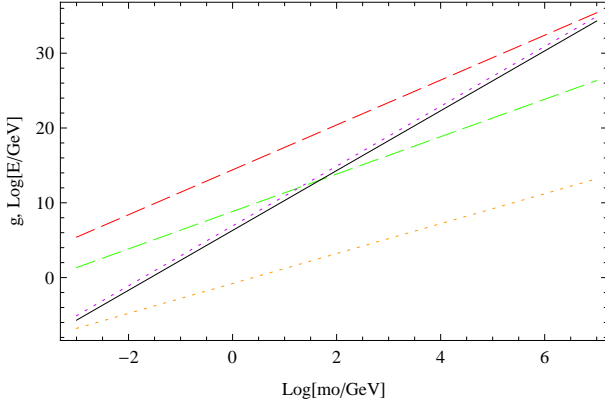


FIG. 3: We show the dependence of the different energies densities as a function of  $m_o$  in  $GeV$ , with  $E_{Rf}$ ,  $E_{Ri}$ ,  $E_h$ ,  $E_g$  and  $E_{dec}$  (dashed-red, dashed-green, dotted-blue, black, dotted-orange, respectively). Since  $E_h \sim E_g$  both lines overlap and we cannot distinguish them.

two for the scalar potential  $V(\phi)$  given by the inflation parameter  $\lambda$  of the quartic term and the mass  $m_o$  given in the quadratic term in  $V$ . The other two parameters are the coupling  $g$  between the inflaton  $\phi$  and a scalar field  $\varphi$  and the coupling  $h$  between  $\varphi$  with standard model particles  $\psi$  or  $\chi$ . These four parameters are the usual ones used previously for a scalar potential and interaction couplings describing the inflaton potential and the reheating process [8]. At the same time, the parameters  $m_o$  and  $g$  are also present in Dark Matter models of WIMP particles. The novel feature here is that without the introduction of any new parameters we are able to unify inflation and Dark Matter using a single field  $\phi$  that accounts for inflation at an early epoch while it gives a Dark Matter WIMP particle at low energies.

We begin with an inflaton field  $\phi$  with a scalar potential

$m_o$	$g^2$	$\Omega_{dm}(E_{Rf})$	$\Omega_{\phi}(E_{Rf})$	$E_{24}$
$10^{-4}$	$1.6 \cdot 10^{-8}$	$4.6 \cdot 10^{-8}$	$3.5 \cdot 10^{-19}$	$1.4 \cdot 10^{-1}$
$10^{-2}$	$1.6 \cdot 10^{-6}$	$4.6 \cdot 10^{-10}$	$3.5 \cdot 10^{-21}$	$1.4 \cdot 10^1$
1	$1.6 \cdot 10^{-4}$	$4.6 \cdot 10^{-12}$	$3.5 \cdot 10^{-23}$	$1.4 \cdot 10^3$
$10^2$	$1.6 \cdot 10^{-2}$	$4.6 \cdot 10^{-14}$	$3.5 \cdot 10^{-25}$	$1.4 \cdot 10^5$
$10^4$	$1.610^0$	$4.6 \cdot 10^{-16}$	$3.5 \cdot 10^{-27}$	$1.4 \cdot 10^7$

TABLE I: We show the values of  $\Omega_{dm}(E_{24})$ ,  $\Omega_{\phi}(E_{24})$  at the scale  $E_{24}$  in  $GeV$  as a function of  $m_o(GeV)$  and the value of the dimensionless coupling  $g^2$ .

$m_o$	$E_{Ri}$	$E_g$	$E_h$	$E_{dec}$
$10^{-4}$	$2.5 \cdot 10^8$	$8 \cdot 10^{-2}$	$2 \cdot 10^{-2}$	$1.6 \cdot 10^{-5}$
$10^{-2}$	$2.5 \cdot 10^{11}$	$8 \cdot 10^2$	$2 \cdot 10^2$	$1.6 \cdot 10^{-3}$
1	$2.5 \cdot 10^{14}$	$8 \cdot 10^6$	$2 \cdot 10^6$	$1.6 \cdot 10^{-1}$
$10^2$	$2.5 \cdot 10^{17}$	$8 \cdot 10^{10}$	$2 \cdot 10^{10}$	$1.6 \cdot 10^1$
$10^4$	$2.5 \cdot 10^{20}$	$8 \cdot 10^{14}$	$2 \cdot 10^{14}$	$1.6 \cdot 10^3$

TABLE II: We show the values of the energies of  $E_R$ ,  $E_d$ ,  $E_{gen}$  and  $E_{dec}$  in  $GeV$  as a function of  $m_o(GeV)$ .

$V$  and interaction term  $V_{int}$ . After a period of inflation our universe must be reheated and we must account for a long period of radiation dominated epoch. Typically the inflaton decays while it oscillates around the minimum of its potential [1]. If the inflaton decay is not complete or sufficient then the remaining energy density of the inflaton after reheating must be either subdominant and play no cosmological role or must be fine tuned to give the correct amount Dark Matter. An essential feature is that the transition between the radiation dominated to the Dark Matter phase is related to a late production of the scalar field  $\phi$ , which takes place naturally without fine tuning, instead to a remanent of Dark Matter energy density which needs to be fine tuned. The scalar potential  $V$  has two terms a quartic and a quadratic one. The quartic term  $V_4 = \lambda\phi^4/4$  gives inflation at high energies  $E_I \simeq 10^{15} GeV$  and via the coupling term  $g^2\phi^2\varphi^2/2$  the large oscillations of  $\phi$  in a preheating stage, with parametric resonances, produce large number of  $\varphi$  particles but since  $\lambda \ll g^2$  the term  $g^2\phi^2\varphi^2/2$  dominates over the potential term  $\lambda\phi^4/4$  and one ends up with  $n_{\varphi} \approx n_{\phi}$ ,  $\varphi^2 \approx \phi^2$  and  $\Omega_{\phi} \approx \Omega_{\varphi}$ . To have a complete decay of the inflaton field, we must couple  $\phi$  or  $\varphi$  to other fields with a trilinear term. Here we use the coupling  $h\varphi\bar{\psi}\psi$  and the massive  $\varphi$  decays into two light  $\psi$  fields. At the same time, this  $\varphi$  decay drags the  $\phi$  field via the chain reaction  $\phi \leftrightarrow \varphi \rightarrow \bar{\psi} + \psi$ . We calculate the end of this process and it has  $\Omega_{\phi} \approx 10^{-23}$  giving a sufficient (efficient) decay. Once  $\phi$  and  $\varphi$  are subdominant the universe has been reheated and we have a standard hot universe filled with relativistic particles. At much lower energies, the same trilinear interaction term  $h\varphi\bar{\psi}\psi$  produce light  $\varphi$  particles, via the reaction  $\bar{\psi} + \psi \leftrightarrow \varphi + \varphi$  below the energy  $E_h = c_d h^4 m_{pl}$ , while the  $g^2\phi^3\varphi^2/2$  term also produces light  $\phi$  particles, through the reaction  $\varphi + \varphi \leftrightarrow \phi + \phi$

below  $E_g = c_d g^4 m_{pl}$  and once the  $\varphi$  have been produced. We see that the coupling terms  $g^2 \phi^2 \varphi^2/2$  and  $h\varphi\bar{\psi}\psi$  play two different rolls namely they allow for  $\phi, \varphi$  to decay and reheat the universe at large energy scales, while they also allow to regenerate these two fields at a much later stages where both fields are relativistic and we have  $n_\varphi = n_\phi$ . As a final step, since the mass of  $\phi$  is different than zero,  $m_o \neq 0$ , the field  $\phi$  will become non-relativistic at  $T \approx m_o$ ,  $n_\phi$  will be exponentially suppressed and  $\phi$  decouples from  $\varphi$ , as any WIMP field. The amount of Dark Matter today fixes the cross section  $\sigma = g^4/32\pi m_o^2$  giving a constraint between  $g$  and  $m_o$ . We show in figs.(2) and (3) the dependence of the  $E$ 's on  $m_o$  and in tables (I) and (II) we give the values for  $10^{-4}GeV < m_o < 10^4 GeV$ .

To conclude, the unification scheme we present here has four free parameters, two for the scalar potential  $V(\phi)$  and two couplings  $g, h$  between the different fields. These four parameters are already present in models of inflation and reheating process, without considering Dark Matter, and in WIMP models without inflation. Therefore, our unification scheme does not increase the number of parameters and it accomplishes the desired unification between inflaton and Dark Matter for free.

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### Appendix A: Summary of Energies

We present the different energy scales relevant in the process of our inflation-Dark Matter unification scheme. Concerning the inflaton-Dark Matter unification scheme we have 4 different parameters  $\lambda, m_o$  in the potential  $V$  and a coupling  $g$  between  $\varphi$  and  $\phi$  and  $h$  the coupling between  $\varphi$  and  $\psi$ .

The energies in eqs.(A2)-(A12) are given in terms of these four parameters. Inflation fixes one parameter,  $\lambda$ , and the amount of Dark Matter today gives a constraint between  $g$  and  $m_o$ . We are left with one single free parameter which we take it to be the mass

$m_o$ . We show in figs.(2) and (3) the dependence of the  $E$ 's on  $m_o$  and in tables (I) and (II) we give the values for  $10^{-4}GeV < m_o < 10^4 GeV$ . We see that  $E_{dec} > E_{eq} = O(eV)$  and that the values of  $g$  and all other energies are phenomenologically viable. This implies that it is feasible to implement the inflation-Dark Matter unification. We resume the definitions and values of these energies

$$g^2 = (32\pi\langle\sigma\rangle)^{1/2}m_o = 1.6 \times 10^{-4} \left(\frac{m_o}{GeV}\right) \quad (A1)$$

$$E_I \equiv \lambda^{1/4}\phi_e = 3 \times 10^{15} GeV \quad (A2)$$

$$E_{Ri} \equiv \rho_{Ri}^{1/4} = \frac{3h^8 g^4 m_{pl}^4}{1024\pi^4 \lambda} \quad (A3)$$

$$= \left(\frac{2h^2}{g^2}\right) \left(\frac{m_o}{GeV}\right)^{3/2} (2.5 \times 10^{14} GeV) \quad (A4)$$

$$\Omega_{\phi Rf} \equiv \frac{64\pi^2 m_o^2}{3g^2 h^4 m_{pl}^2} \quad (A5)$$

$$= \left(\frac{g^2}{2h^2}\right)^2 \left(\frac{GeV}{m_o}\right) 3 \times 10^{-23} \quad (A6)$$

$$E_{Rf} \equiv \rho_{Rf}^{1/4} \equiv \left(\frac{3g^2 h^4 m_o^2 m_{pl}^2}{32\lambda\pi^2}\right)^{1/4} \quad (A7)$$

$$= \sqrt{\frac{2h^2}{g^2}} \left(\frac{m_o}{GeV}\right)^{5/4} 6.8 \times 10^8 GeV. \quad (A8)$$

$$E_h \equiv c_d h^4 m_{pl} \quad (A9)$$

$$= \left(\frac{2h^2}{g^2}\right)^2 \left(\frac{m_o}{GeV}\right)^2 2 \times 10^6 GeV$$

$$E_g \equiv c_d g^4 m_{pl} \quad (A10)$$

$$= \left(\frac{m_o}{GeV}\right)^2 8 \times 10^6 GeV$$

$$E_{dec} \equiv c_{dec} T_F = \left(\frac{\pi^2 g_{rel}}{30}\right)^{1/4} \frac{m_o}{x_F} \quad (A11)$$

$$= 0.1 \left(\frac{10}{x_F}\right) \left(\frac{m_o}{GeV}\right) GeV$$

$$E_{24} \equiv \left(\frac{m_o^2 \phi_{24}^2}{2}\right)^{\frac{1}{4}} = \left(\frac{4}{\lambda}\right)^{1/4} m_o \quad (A12)$$

$$= 1.4 \times 10^3 m_o.$$

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